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ANALYSIS OF ALGORITHMS FOR SOLVING TWO MACHINE FLOW SHOP SCHEDULING PROBLEMS – HEURISTIC APPROACH

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Abstract

This paper studies scheduling problem of two-machine flow shop (FSSP). Minimization of maximum completion time always leads to the improvement of all the relevant measures of flow lines. Minimization of total completion time also much important in the production environment. In our earlier work we had developed two new heuristics based on Johnson's rule for solving FSSP with makespan objective which reduces the total completion time criteria also. In this paper, we have extended the investigation of our algorithms to a objective which combines both the objectives. Through case studies, we showed that one of our algorithms performed well than the other two.

Keywords: Scheduling problem, flow shop, makespan, total completion time, production scheduling.

1. INTRODUCTION

In production scheduling problem, FSSP is one of the most important field on which many of the objectives has been investigated by many researchers for decades. In flow shop environment, all the jobs are to be executed by a series of machines in the same technological order. Since Johnson [1] compiled a famous heuristic rule for makespan objective, a large number of researchers developed heuristic and meta heuristic to enhance the quality or performance analysis of their algorithms for the different objectives in variety of situations. Indeed, many of the papers resulting the same general performance but produces different results precisely, global researchers have been working in this best known scheduling problem for the development of new heuristic or for the improvement of heuristic and meta heuristic. Different constraints imposed on the different situations brief the necessity of the above said heuristics. Makespan (C_{max}) is one of the main objectives of FSSP which is the completion time of the last job on last machine. In deterministic scheduling problems, minimization of average completion time is same as the minimization of total completion time (TCT) ($\sum C_j$) where C_j denotes the completion time of job j. The inconsistency of these two main objectives $\min C_{max}$ and $\min \sum C_j$ of FSSP is addressed by Li et.al. [2] in 2015 and to resolve this inconsistency, they introduced a new objective

$$\min \left(\sum_{i=2}^m C_{max,i} + \sum_{i=1}^m \sum_{j=1}^n C_{i,j} \right) \quad (1)$$

for the n jobs m machine FSSP. This new approach includes the

overall performance of $\min C_{max}$ and minimization of $\sum C_j$ on every machine which is more useful than individual objectives. We have taken into consideration of this objective for the two machine case in this paper when the machine order is reversed.

2. LITERATURE REVIEW

A large amount of research has been done on FSSP which is formulated initially by Johnson in the mid of 1954. After him, many researchers were developed their idea based on Johnson's rule and found new heuristic to obtain better results or to make an improvement of the old heuristics. We wish to introduce some of the famous heuristic methods such as Palmer's Slope index method, Cambell et al. CDS method, Gupta's slope index method, Dannenbring's Rapid Access heuristic algorithm and NEH heuristic algorithm which were developed based on Johnson's rule. In 1965, Palmer [3] proposed a method of slope index which assign a number to each job and sorting the jobs according to their slope index. In 1970, Cambell et. al. [4] developed a heuristic algorithm which applies Johnson's rule in each of $m-1$ stages for m machine FSSP. The $m-1$ two machine problem were generated from the original problem and then Johnson's rule was applied in all the stages. A schedule was chosen from the out of $m-1$ schedules which gives best result. In 1971, Gupta [5] proposed another slope index method which is similar to Palmer's. In 1977, Dannenbring's [6] developed a rapid access heuristic method by mixing Johnson's rule and Palmer's slop index method. He introduced a weighting scheme to each machine after creating two virtual machine as in CDS

method then utilized Johnson’s rule to find the required sequence. In 1983, Nawaz et.al.[7] developed a heuristic called NEH heuristic which was considered best for makespan criterion for permutation FSSP. Several heuristics and improvement methods have been created by many researchers which ensure the importance and need of a new algorithm for different conditions. In [8], Jayakumar et. al. proposed an algorithm for solving FSSP to minimize the makespan which provides better results than CDS method. In [9], Jayakumar et. al. were proposed a new algorithm for the 2 machine n jobs FSSP and compared the results with Johnson’s rule when the machine order is reversed. In [10, 11], Jayakumar et. al. considered the open shop scheduling problem with TCT objective for the general and hypothetical situations. In [12, 13] Sathiya Shanthi et.al. addressed the two machine and three machine cases for the minimization of C_{max} while in [14] Sathiya Shanthi et.al. investigated about the minimization of $\sum C_j$ for the two machine FSSP. This motivates us to consider the objective in (1) to analyze our developed algorithms along with Johnson’s rule. In this paper, we have been discussing the performance of the objective in (1) for the two machine n jobs FSSP.

3. METHODOLOGY

In a FSSP, a set of n - jobs has to be processed on 2 - different machines in the same order. Each job $j, j=1,2,\dots,n$ must process on machines A and B with the non negative processing times A_1,A_2,\dots,A_n and B_1,B_2,\dots,B_n . Each machine can processes at most one job and each job can be handled at most one machine at any given time. Once the job started its processing, it should be completed without any interruption. The machines process the jobs in a first come first served manner. The jobs are processed on machine B first followed by machine A (i.e., in the order B – A). Johnson algorithm breaks the tie situation occur on same machines by process the job with smallest index. In case of tie occur in between A_j, B_j , it can be broken by process the job which falls on machine A_i while in CDS method, the tie situation was broken by giving priority to the job in the same position of the previous stage. For handling this tie criterion, we presented two priority rules and two heuristic algorithms namely Sathiya Shanthi Meganathan Jayakumar – Shortest Processing Time (SMJ-SPT) and Sathiya Shanthi Meganathan Jayakumar – Largest Processing Time (SMJ-LPT) which can be found in [9, 12]. For a given two machine FSSP, we obtained three different sequences by applying three algorithms such as Johnson rule and our proposed algorithms. The illustration of the various sequence obtaining methods are explained in the next section in detail.

4. ILLUSTRATION

Consider a FSSP with 6 - jobs on 2 - machines.

Machines/ Jobs	1	2	3	4	5	6
M/C A	19	17	32	31	28	21
M/C B	21	51	19	17	27	20

Now this two machine FSSP can be solved by three different

algorithms for the objective of minimization of the new objective in equation (1). Let us start with applying Johnson’s rule at first. Observe that the smallest processing time is 17 which fall in two places such as for job 2 on machine A and for job 4 on machine B. So the tie can be broken arbitrarily such that job 2 can be scheduled first and job 4 can be scheduled last in the sequence. The next smallest processing time is 19 again which fall in two places as for job 1 on machine A and for job 3 on machine B. So this tie can be broken arbitrarily such that job 1 can be scheduled after job 2 and job 3 can be scheduled before job 4 in the sequence. The remaining 2 jobs can be scheduled as usual and we obtain a sequence as follows:

2	1	5	6	3	4
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Johnson’s rule

If we apply SMJ-SPT algorithm for the above problem, the first tie situation can be handled by applying priority rule 1 and after observing the processing times of job 2 and job 4 on the respective other machines, we found that the smallest processing time is 31 which fall in machine A for job 4 and so we schedule job 4 first in the sequence and job 2 after job 4. Similarly in the second tie situation, the smallest processing time is 21 which fall in machine

B for job 1 and so we schedule job 1 last in the sequence. After all the jobs are scheduled, the sequence for the SMJ-SPT algorithm is as follows:

4	2	3	5	6	1
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SMJ-SPT Algorithm

If we apply SMJ-LPT algorithm for the above problem, the first tie situation can be handled by applying priority rule 2 and after observing the processing times of job 2 and job 4 on the respective other machines, we found that the largest processing time is 51 which fall in machine B for job 2 and so we schedule job 2 last in the sequence and job 4 before job 2. Similarly in the second tie situation, the largest processing time is 32 which fall in machine A for job 3 and so we schedule job 3 first in the sequence. After all the jobs are scheduled, the sequence for the SMJ-LPT algorithm is as follows:

3	1	5	6	4	2
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SMJ-LPT Algorithm

Table 1: Time in Time out table for the sequence 3-1-5-6-4-2

Jobs	M/C B		M/C A	
	In	Out	In	Out
3	0	19	19	51
1	19	40	51	70
5	40	67	70	98
6	67	87	98	119
4	87	104	119	150
2	104	155	155	172
Total		472		660

Table 1 showed that the completion times of the sequence obtained by SMJ-LPT algorithm. For the sequence 3-1-5-6-4-2, the maximum completion time on machine B and machine A are

155 and 172 respectively also total completion times are 472 and 660 on machine B and machine A respectively. Thus the performance of the objective in equation (1) is $172+472+660=1304$. Similarly for the sequence 2-1-5-6-3-4 obtained by applying Johnson's rule, the performance of the objective in equation (1) is $211+634+825=1670$ for the sequence 4-2-3-5-6-1 obtained by applying SMJ-SPT algorithm, the performance of the objective in equation (1) is $187+575+754=1516$. From this we conclude that SMJ-LPT algorithm outperforms well than the other two algorithms for this problem.

5. COMPUTATIONAL RESULTS

To test the performance of the equation (1) for our proposed heuristic algorithms in two machine cases, the job number is chosen from 4, 5, 6, 7, 8, 9, 10. Consequently there are 7 scales of instances and there are 10 instances for each scale and 70 problem instances in total. The processing times are chosen arbitrarily from the numbers 1 to 99. Sum of the values of equation (1) obtained for 10 instances for each problem size is presented in Table 2. From that, it is observed that our proposed SMJ-LPT algorithm provides better results than Johnson's rule as well as SMJ-SPT algorithm. It is further observed that for the performance of equation (1), SMJ-SPT algorithm outperforms well than Johnson's rule in 37 out of 70 problem instances were tested and SMJ-LPT algorithm outperforms well than Johnson's rule in 62 out of 70 problem instances were tested. Out of 70 problem instances were tested, SMJ-LPT algorithm provides best result than SMJ-SPT algorithm in 46 problem instances, equal values in 2 problem instances and worst result in 22 problem instances.

Table 2: Sum of the values of the objective in equation (1)

m	n	Johnson's rule	SMJ-SPT algorithm	SMJ-LPT algorithm
2	4	8977	8664	7814
2	5	15314	15054	14061
2	6	13886	12463	11751
2	7	32042	31203	26218
2	8	27687	26630	23129
2	9	29688	29024	27902
2	10	51088	50166	47623
Total		178682	173204	158498

We wish to test the significance of our analysis in two different pairs such as Johnson's rule against SMJ-SPT algorithm and Johnson's rule against SMJ-LPT algorithm. The corresponding results are shown in Table 3. Each test size has 10 problem instances and gives ten total completion time values and we thus have a paired comparison test. The mean and standard deviation are obtained for each test size. The difference is calculated by subtracting the completion time of the proposed heuristic from that of Johnson's algorithm. Now set the hypothesis for the population corresponding to the difference has mean μ equal to zero. Here we test the null hypothesis μ equal to zero against the alternative μ greater than zero. Also we

suppose that the difference of the completion times is a normal variable and assume the significance level $\alpha = 0.05$. If the hypothesis is true, the random variable $t = \sqrt{N} \left((\bar{X} - \mu_0) / S \right)$ has a t-distribution with $N - 1$ degrees of freedom, where N denotes sample size, \bar{X} denotes sample mean, S denotes sample standard deviation and $\mu_0 = 0$. The critical value c is obtained from the relation probability $(t > c) = \alpha = 0.05$. From the standard table values of t-distribution, we observed that the table value of c for 9 degrees of freedom is 1.83. Table 2 has shown that in most of the cases SMJ-SPT algorithm or SMJ-LPT algorithm or both are significantly better than Johnson's rule statistically.

Table 3: Performance of Johnson's, SMJ-SPT and SMJ-LPT Algorithms

m	n	Difference of Johnson and SMJ-SPT		t	Difference of Johnson and SMJ-LPT		T
		Mean	SD		Mean	SD	
2	4	31.3	140.74	0.67	116.3	97.78	3.57
2	5	26	122.67	0.64	123	115.68	3.25
2	6	142.3	153.98	2.77	213.5	317.66	2.02
2	7	83.9	393.54	0.64	582.4	445.98	3.92
2	8	105.7	394.95	0.8	455.8	295.84	4.62
2	9	66.4	324.88	0.61	178.6	293.13	1.83
2	10	92.2	568.41	0.49	346.5	292.39	3.56

6. CONCLUSION

In manufacturing systems, minimization of production line utilization related to makespan which is more important objective of FSSP. Moreover minimization of work in process inventories is also related to TCT. These two common objectives of FSSP are the main concerns of many researchers over the years. In this paper, we analyzed three algorithms for the performance of the objective in equation (1) and results shows that our SMJ-LPT algorithm performs well for the two machine case. Many real life disturbances in the production line control with various constraint environments may be addressed for this objective to achieve adoptive production control.

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